



Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

NOTES AND QUERIES.

WE have received the following from Mr. William Orchard, F.I.A.:—

PETERSBURG PROBLEM.—This is the name of a celebrated problem in the Theory of Probabilities, which has given rise to much discussion.

The problem is this:—A coin is thrown until head appears: if it appear at the first time, A is to pay B £2; if at the second, £4; if at the n th throw, £2 n . What should B pay to A before the commencement of the game for his expectation of gain by it?

The probability of head appearing the first time is $\frac{1}{2}$; the gain if it appear is £2; therefore the value of B's expectation upon this throw is £1; and, generally, the value of his expectation upon the n th throw is £1; for that the n th throw may occur, head must not have appeared in the first $(n-1)$ throws, the probability of which is $\frac{1}{2^{n-1}}$; that it will then appear at the following throw is $\frac{1}{2}$, and the consequent gain £2 n : therefore his expectation is $\frac{1}{2^{n-1}} \cdot \frac{1}{2} \cdot 2^n = 1$. If the game is to be continued until head appears, however long it may be deferred, B's expectation of gain is infinite; for an infinite number of throws are possible, and must be allowed for in the calculation.

Although such is the strict mathematical conclusion, no one, not even a mathematician, would give much for the expectation of gain which the game offers. Daniel Bernoulli, in discussing the problem in the *Petersburg Memoirs*, whence its name, invented a theory of moral expectation, distinguished from mathematical expectation by the consideration that the value of a sum of money to an individual depends upon the amount of his previous fortune. By means of this theory,—for which, and its application to the present problem, I refer to the *Essay on Probability*, L. U. K., De Morgan's Treatise on Probability in the *Encyclopædia Metropolitana*, and Galloway's Treatise from the *Encyclopædia Britannica*,—he deduced a finite and very small value for B's moral expectation, depending upon the amount of his fortune, but not at all upon that of A.

Poisson has, in his *Recherches sur la Probabilité des Jugements*, given a very happy solution of the problem, without having recourse to the theory of moral expectation. As this solution has not appeared in any English treatise, I lay a sketch of it before your readers, referring to Poisson's own work for more detail and generality.

Let A's fortune be £2 n , then if head appear in the first n throws, A will be able to pay the whole loss: B's expectation upon these n throws is £ n . But if head be not thrown until after n throws, however much B may gain by the conditions of the game, he can only receive from A the whole amount of his fortune, that is, £2 n . From this cause B's expectation on all the throws after the n th is reduced to—

$$2^n \left\{ \frac{1}{2^{n+1}} + \frac{1}{2^{n+2}} + \dots \right\} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots = 1;$$

and therefore the value of B's entire expectation is $(n+1)$ pounds.

Thus, if A's fortune were so large as $2^{20} = 1,048,576$ pounds, the sum which B would be justified in paying him for the possible gains under the conditions of the problem would be but £21.